

# Semileptonic $B$ decays and the inclusive determination of $|V_{ub}|$

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**Abstract.** We present a new theoretical framework for the study of  $B \rightarrow X_u \ell \nu$  decays, which includes all known perturbative and non-perturbative contributions and a description of leading and subleading Fermi motion effects. The perturbative and non-perturbative regimes are separated by a “hard” Wilsonian cutoff  $\mu \sim 1$  GeV. We bring into focus some problems related to the high  $q^2$  region and to Weak Annihilation effects. We provide estimates of the CKM parameter  $|V_{ub}|$  using the described framework and discuss the related theoretical uncertainty.

## 1. Introduction

The precise determination of the element  $|V_{ub}|$  of the CKM matrix is an important test of the flavour structure of the Standard Model (SM) and is crucial in the indirect search for New Physics. The latest global fit to the Unitarity Triangle (UT) including all flavour changing observables but a direct determination of  $|V_{ub}|$  predicts  $|V_{ub}| = (3.44 \pm 0.16) \times 10^{-3}$  [1]. This value agrees within errors with the *exclusive* determination, that relies on lattice QCD or light-cone sum rules [2, 3] and that is still affected by somewhat large theoretical errors. An *inclusive* analysis is in principle the cleanest method to precisely determine  $|V_{ub}|$ . This is based on the comparison between the decay rate of  $B \rightarrow X_u \ell \nu$  measured by experiments and the corresponding theoretical prediction. The latest HFAG world average [4] yields an inclusive  $|V_{ub}|$  which is about  $2.5\sigma$  higher than the value preferred by the global UT fit, calling for a deeper investigation of the process.

The theoretical description of  $B \rightarrow X_u \ell \nu$  is based on a local Operator Product Expansion. Inclusive quantities are organized in a double series in  $\alpha_s$  (perturbative QCD corrections) and in  $1/m_b$  (Heavy Quark Expansion). The very same method was successfully applied to the  $b \rightarrow c$  decay and led to a precise determination of  $|V_{cb}|$ , within 2%. The description of charmless decays is more involved due to the dominant charmed background that needs to be rejected by experiments imposing very stringent cuts. These cuts can spoil the convergence of the OPE introducing sensitivity to nonlocal effects, such as the motion of the  $b$  quark inside the meson (Fermi motion), that can be parameterized in terms of a light-cone distribution function (or “shape function”). The lowest integer moments of the distribution function are constrained by the OPE [5] and they are expressed in terms of the  $b$  quark mass and of the same 5 and 6 dimensional operators that contribute to  $B \rightarrow X_c \ell \nu$ . Such expressions are universal, i.e. independent of the process, and shared by the radiative decay  $B \rightarrow X_s \gamma$  only as long as  $1/m_b$  corrections are neglected.

An OPE-based treatment of shape function effects in  $B \rightarrow X_s \gamma$  including subleading ( $1/m_b$ ) effects was developed in [7] and turned out to describe well experimental data. In [8] a similar procedure was undertaken for the case of semileptonic decays, where many complications arise, mostly due to the kinematics taking place at different  $q^2$ . In the following we illustrate the main features of this procedure and show some meaningful results.

## 2. Theoretical framework

### 2.1. Perturbative corrections in a Wilsonian approach

All observables describing the  $B \rightarrow X_u \ell \nu$  decay can be extracted *via* integration over the triple differential width:

$$\begin{aligned} \frac{d^3\Gamma}{dq^2 dq_0 dE_\ell} &= \frac{G_F^2 |V_{ub}|^2}{8\pi^3} \left\{ q^2 W_1 - \left[ 2E_\ell^2 - 2q_0 E_\ell + \frac{q^2}{2} \right] W_2 + q^2 (2E_\ell - q_0) W_3 \right\} \times \\ &\quad \times \theta \left( q_0 - E_\ell - \frac{q^2}{4E_\ell} \right) \theta(E_\ell) \theta(q^2) \theta(q_0 - \sqrt{q^2}), \end{aligned} \quad (1)$$

where  $q_0$  and  $E_\ell$  are the total leptonic and the charged lepton energies in the  $B$  meson rest frame,  $q^2$  is the leptonic invariant mass and  $W_{1-3}$  are the three structure functions relevant in the case of massless lepton.

Perturbative corrections to the structure functions  $W_{1-3}$  to order  $\mathcal{O}(\alpha_s)$  have been known for quite long [9], whereas  $\mathcal{O}(\alpha_s^2 \beta_0)$  corrections recently appeared in [10]. Both calculations were performed in the *on-shell* scheme.

It has been stressed several times in the literature [11] that an *on-shell* definition of the  $b$  quark mass becomes ambiguous as soon as power suppressed terms are included. The *pole* mass is better traded with a *running* mass  $m_b(\mu)$ . To this purpose, perturbative corrections to order  $\mathcal{O}(\alpha_s^2 \beta_0)$  are calculated anew in [8] in the presence of a “hard” Wilsonian cutoff  $\mu \sim 1$  GeV. This new scale separates the perturbative regime of gluons with energies larger than  $\mu$  from the “soft” (non-perturbative) regime of gluons with energies lower than  $\mu$ . The contributions of soft gluons are then absorbed into a redefinition of the heavy quark parameters, consistent with the way they are extracted from fits to  $B \rightarrow X_c \ell \nu$  moments in the *kinetic* scheme [12, 6]. Physical observables are of course independent of the cutoff.

### 2.2. Fermi motion

As already mentioned in the Introduction, Fermi motion is encoded in a distribution function, whose lowest integer moments are constrained by the local OPE. As soon as  $1/m_b$  corrections are retained, such moments cease to be universal: they have different expressions for each of the three structure functions in eq. (1) and show an explicit  $q^2$  dependence. To preserve generality we introduce three separate distribution functions, one for each of the structure functions, depending on the light-cone component of the  $b$  quark momentum ( $k_+$ ), on  $q^2$  and on the Wilsonian cutoff ( $\mu$ ). Hadronic structure functions are then defined *via* a convolution of the perturbative structure functions with the distribution functions, whose expression is derived at the leading order in  $1/m_b$  and  $\alpha_s$  and assumed to be valid also at higher orders:

$$W_i(q_0, q^2) \sim \int dk_+ F_i(k_+, q^2, \mu) W_i^{pert} \left[ q_0 - \frac{k_+}{2} \left( 1 - \frac{q^2}{m_b M_B} \right), q^2, \mu \right] \quad (2)$$

Model-dependence resides only in the *Ansatz* employed for the distribution functions. In the analysis of [8] a set of about 80 different functional forms, inspired by those already present in the literature (exponential, Gaussian, Roman, hyperbolic), is tested and the related uncertainty on  $|V_{ub}|$  turns out to be rather small (see Sec. 3).

cuts	$ V_{ub}  \times 10^3$	$f$	exp	par	pert	tail model	$q_*^2$	$X$	ff	tot th
<b>A</b> [15]	3.87	0.71	6.7	3.5	1.7	1.6	2.0	$+0.0$ $-2.7$	$+2.4$ $-1.1$	$\pm 4.7^{+2.4}_{-3.8}$
<b>B</b> [15, 16]	4.44	0.38	7.3	3.5	2.6	3.0	4.0	$+0.0$ $-5.0$	$+1.4$ $-0.5$	$\pm 6.6^{+1.4}_{-5.5}$
<b>C</b> [17]	4.05	0.30	5.7	4.2	3.3	1.8	0.9	$+0.0$ $-6.2$	$+1.2$ $-0.7$	$\pm 5.7^{+1.2}_{-6.9}$

**Table 1.** Values of  $|V_{ub}|$  obtained using different experimental results and their experimental and theoretical uncertainties (in percentage) due to various sources (see text).  $f$  is the estimated fraction of events.

### 2.3. The high $q^2$ region

The impact of Fermi motion becomes irrelevant at high  $q^2$  and the developed formalism is no more applicable. The OPE itself shows a number of pathological features in this kinematical region, due to the emergence of unsuppressed higher order terms. For instance, the OPE predicts a value for the variance of the distribution functions which decreases at increasing  $q^2$ , reaching even negative values. Moreover, Wilson coefficients of power suppressed operators become more and more important and already the coefficient of the Darwin term ( $\rho_D^3$ ) shows a divergence at  $q^2 = m_b^2$ :

$$\frac{d\Gamma}{d\hat{q}^2} \sim \frac{\rho_D^3}{6m_b^3} \left[ 20\hat{q}^6 + 66\hat{q}^4 + 48\hat{q}^2 + 74 - \frac{96}{1-\hat{q}^2} \right] + \dots, \quad \hat{q}^2 = \frac{q^2}{m_b^2} \quad (3)$$

This singularity is removed at the level of the total rate by a one-loop penguin diagram that mixes the Weak Annihilation (WA) four-quark operator into the Darwin operator [13, 14]. However, as we are interested in differential distributions as well, a dedicated treatment of the high- $q^2$  region ( $q^2 > q_*^2 \sim 11 \text{ GeV}^2$ ) is mandatory. In [8] two different methods are proposed and their difference is used to estimate the associated uncertainty:

- a) we model the tail in a way consistent with positivity of the spectra and including a WA contribution ( $X$ ) through a Dirac- $\delta$  localized at the endpoint (default method):

$$\frac{d\Gamma}{d\hat{q}^2} \sim \frac{\rho_D^3}{6m_b^3} \left[ 20\hat{q}^6 + 66\hat{q}^4 + 48\hat{q}^2 + 74 - \frac{96(1 - e^{-\frac{(1-\hat{q}^2)^2}{b^2}})}{1-\hat{q}^2} \right] + X \delta(1-\hat{q}^2) + \dots \quad (4)$$

- b) we extend the Fermi motion description of low  $q^2$  to the high  $q^2$  region, *freezing* the shape function at  $q^2 = q_*^2$  and using it in the convolution of eq. (2) at higher  $q^2$ .

## 3. Results and theoretical uncertainties

We take advantage of some of the latest experimental measurements to extract values of  $|V_{ub}|$  using the described framework. We leave the task of an average of these results to a future, hopefully dedicated, experimental analysis. We consider:

- A** Belle analysis with  $M_X \leq 1.7 \text{ GeV}$  and  $E_\ell > 1.0 \text{ GeV}$  [15];
- B** Belle and Babar analyses with  $M_X \leq 1.7 \text{ GeV}$ ,  $q^2 > 8 \text{ GeV}^2$ , and  $E_\ell > 1.0 \text{ GeV}$  [15, 16];
- C** Babar analysis with  $E_\ell > 2.0 \text{ GeV}$  [17]

Results are summarized in Table 1 and were obtained from a C++ code available upon request. The reported values of  $|V_{ub}|$  are obtained using the default setting, namely an exponential *Ansatz* for the distribution functions, the prescription a) for the high  $q^2$  tail at  $X = 0$  (see previous section) and the central values of the fit in [6] as input parameters, at  $\mu = 1 \text{ GeV}$ .  $f$  is the estimated fraction of events, whereas the following columns show different sources of uncertainty, namely:

- Experimental error (exp).
- Parametric error (par): it is extracted taking into account all correlations between non-perturbative parameters [6] and varying  $\alpha_s = 0.22$  by  $\pm 0.02$  as uncorrelated. The uncertainty on  $m_b$  is by far dominating.
- Perturbative error (pert).
- Errors related to the high  $q^2$  region: we consider the difference between methods *a*) and *b*) (tail model), the value of  $q^2$  at which the modelling sets in ( $q_*^2$ ) and the error due to WA effects ( $X$ ). We let  $X$  vary in a range consistent with the 90% confidence level bound set by CLEO on the size of WA [18], namely  $0 \leq X \leq 0.04$ . We stress that the error related to  $X$  is asymmetric and points to a lower value of  $|V_{ub}|$ .
- Functional form dependence (ff), estimated using about 80 different *Ansätze* for the distribution functions.

It is worth stressing that all combinations of cuts considered include the high  $q^2$  region discussed in the previous section which, as we showed, is plagued by poorly controlled effects. However, Belle measurements **A** and **B** can be easily combined in order to obtain an estimate of  $|V_{ub}|$  with an *upper* cut on  $q^2$ , namely for the combination  $M_X \leq 1.7 \text{ GeV}$ ,  $E_\ell > 1.0 \text{ GeV}$ , and  $q^2 < 8 \text{ GeV}^2$ . This yields, within our framework, a value of  $|V_{ub}|$  much lower than in all other cases ( $|V_{ub}| = 3.18 \times 10^{-3}$ ), and it might signal some bias in the treatment of the high  $q^2$  region either on the experimental or on the theoretical side. Only a dedicated experimental analysis with an upper cut on  $q^2$  could probably shed some light on this issue.

#### 4. Summary and References

We presented a new approach for dealing with the triple differential width of  $B \rightarrow X_u \ell \nu$  decays, in a framework characterised by a hard Wilsonian cutoff  $\mu \sim 1 \text{ GeV}$ . The method developed takes into account all known perturbative and non-perturbative corrections. Fermi motion is treated at the subleading level as well. Some problems related to the high  $q^2$  region of the process were pointed out, that were probably underestimated in the past and that still deserve a deeper investigation. We also presented some numerical results with the associated uncertainties and put forward the suggestion of a new experimental analysis with an upper cut on  $q^2$ .

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